Technical Comments

Comment on "Thermal Instability of a Viscosity Stratified Fluid Layer Heated from Below"

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THE paper by Chandra[1] is misleading in two respects. First, the author concludes that "if δ is positive, i.e., the viscosity is increasing upwards, the critical Rayleigh number is increased," which might lead the reader to think that some sort of stabilization process is involved. In fact, the author's results imply that the criterion for instability is unchanged, if the Rayleigh number is defined in terms of the mean viscosity (rather than the viscosity at the lower boundary).

The paper is also misleading in its neglect of virtually all of the literature on this subject published since 1955. Particularly pertinent here are the papers by Palm[2] and Jenssen,[3] who have carried out a calculation to a higher order of approximation than that found in Chandra's work, and have shown that variation of viscosity leads to a reduction in the critical Rayleigh number based on the mean viscosity.

In fact, the more interesting effects due to variation in viscosity are not brought out by linear stability analysis, but require nonlinear analysis. For a discussion of these effects, the reader is referred to Sec. 77 of the work by Joseph,[4] the paper by Booker,[5] and the references contained herein.

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Comment on "Extensions of Dual-Plate Holography Interferometry"

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THE authors[1] describe the application of dual-plate holographic interferometry to the flow over a cone. Figure 3 of their paper illustrates the comparison between analytical and experimental results. The agreement provides confidence in quantitative capability of holographic interferometry.

Similar accuracy was obtained ten years ago without the complexity of dual-plate holography. One example is reported in the Note by Holds and Fuhs[2]; this example is for a gas having $\gamma = 1.67$. Another example is reported in the paper by Fuhs.[3] The flow was in air at Mach 2.7 with a 40 deg cone.

For simple flow geometries such as conical flow, the complexity of dual-plate holography may not be necessary. However, for flows with more structure, e.g., Figs. 4 and 5 of Ref. 1, dual-plate holography offers distinct advantages.

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[1] Hannah, B. W. and King, W. L., Jr., "Extensions of Dual-Plate Holography Interferometry," *AIAA Journal*, Vol. 15, May 1977, pp. 725-727.

[2] Holds, J. H. and Fuhs, A. E., "A Refined Analysis of a Holographic Interferogram," *Journal of Applied Physics*, Vol. 38, 1967, pp. 5408-5409.

[3] Fuhs, A. E., "Plasma Diagnostic Techniques," Paper 7-1 in AGARD *Conference Proceedings*, No. 38, Technivision Services, Slough, Bucks, England, 1970; also presented at the 1967 PEP AGARD Meeting.

Reply by Authors to A. E. Fuhs

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FUHS is correct in stating that "for simple flow geometries such as conical flow, the complexity of dual-plate holography may not be necessary." We would like to restate from Ref. 1 what we believe to be the key to the necessity for dual-plate (i.e., movable fringe) capability. "One of the main problems in quantitative production applications of interferometry is the basic lack of continuous data in conventional interferometric images. In any given image, only a limited number of fringes cross any line along which quantitative data are desired. The ideal situation would be the generation of a data curve giving fringe shift as a continuous function of position in the flow, but the number of data points in any single image precludes this possibility."

In the strictest sense this is not always true because, for very special flow cases (i.e., conical flows, etc.), fringe shift data from an entire image can be plotted on one curve due to the similarity of fringe shape as well as flowfield. Reference 2 demonstrates this "similar" property of fringe shape by plotting g/l (fringe shift function normalized by axial length at station of measurement) against R/r_c (radial position in flowfield, at axial station of measurement, normalized by body radius, at axial station of measurement). By employing this scaling, the fringe shift data from an entire conical flowfield can be plotted on one curve. In point of fact, Ref. 3,

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mentioned by Prof. Fuhs, employs this symmetry to generate the theoretical fringe shift patterns used in his data comparisons with the interferograms of Ref. 4.

It seems appropriate here to state the system requirements which led to the use of the dual-plate technique in our application: 1) applicability to arbitrary flow and model shapes, 2) the ability to generate accurate data on flowfield density distributions from the interferograms of experimentally produced flowfields; and 3) the ability to interface the interferometer technique with an automated image reading and processing system. On point 1 we are in agreement with Prof. Fuhs concerning the advantages of the dual-plate technique for complex flowfields.

On point 2 there should be an explanation of the differences in method employed in our work and the references cited by Fuhs. Our methodology[1] is general in nature and is directly applicable to any axisymmetric flowfield. In fact, the experimental techniques for accurate generation of fringe shift function could be applied to three-dimensional flows per the methods of Ref. 2. Figure 2 of Ref. 1 shows the measured fringe shift function for our sample flowfield, a cone at zero angle of attack. This flow example was chosen for the ease of analysis of the theoretical flowfield, whereas our experimental data analysis did not employ the "similar" property of the fringe pattern and was therefore quite general in methodology.

The second step in our data reduction process was the conversion of the fringe shift function to flowfield density. It is here that significant differences exist betweeen the methodology of Ref. 1 and those cited by Prof. Fuhs. If, as in Ref. 1 the flow of data is from the fringe shift function to the calculation of flowfield density (the data flow needed for application to flowfield measurement) the data reduction theory (for axisymmetric flow) contains a derivative of the fringe shift function. The form of the equation is

$$n(r) - n_0 = -\frac{\lambda}{\pi}\int_r^R \frac{S'\,\mathrm{d}y}{\sqrt{y^2 - r^2}} \tag{1}$$

where n is the index of refraction related to flowfield density by the Dale-Galdstone relation and S' is the derivative of measured fringe shift function with respect to radial position in the flowfield. Reference 5 enumerates the techniques used to evaluate this integral. It is this facet of the general data reduction methodology which required, even in its application to conical flow, the use of the dual-plate technique. Without the dual-plate technique, the previously mentioned lack of data points from a single image would introduce large errors in the derivative of the measured fringe shift function. These errors would then translate directly into errors in measured flowfield density. In Ref. 1 any single image provided five data points to define the entire function, thereby providing poor definition of S'. With the use of movable fringes, however, 23 data points were taken, sufficient to provide the accuracy shown in Fig. 3 of Ref. 1. In the references cited by Fuhs the flow of data was from theoretical density distribution to the generation of theoretical fringe pattern. In this methodology the theoretical density distribution is converted, by the Dale-Galdstone relation, into the theoretical distribution of index of refraction. The resultant fringe shift function is calculated from

$$s(x,y) = \frac{2}{\lambda}\int_y^R [n(r,x) - n_0]\,\frac{r\mathrm{d}r}{\sqrt{r^2 - y^2}} \tag{2}$$

This form can be evaluated much more accurately than can Eq. (1) because it does not contain the derivative of an experimentally generated function.

Two comments now appear in order. First, although our work does not by any means represent the first successful generation of flowfield density data from interferograms (Refs. 6, 7, etc.), it does represent the potential for greatly increased accuracy and versatility in applying interferometry to a wide variety of complex flowfields. Second, the methodology cited by Prof. Fuhs is sound as a measure of interferometer performance and has been employed to check system performance for dual-plate applications.[5] However, it does not represent similar accuracy to that shown in Ref. 1 in that it does not generate measured flowfield density from experimental data.

In closing we would like to mention the suitability of the dual-plate method to the third system requirement, interfacing with an automated image reading and processing system. With the use of the scene plate alone, very sharp images, void of fringes, can be obtained for accurate determination of body position by automated means. In addition shadowgraphs of any sensitivity or schlierens of any knife edge orientation can be obtained on reconstruction, providing additional flowfield information. This aspect of the technique (i.e. automated processing) is now under development and will be reported in the future.

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